Chapter 9, BigFraction

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Contents

0 A Brief Introduction to Sets

The purpose of this reading is to convey sufficient knowledge of relations and ordering so the student can proceed intelligently in dealing with these issues in the context of data structures.

We will assume a very basic knowledge of boolean operations, and we will introduce a little set theory here.

Every disucssion we take part in about collections involves a *universe of discourse*. This is the set of all things we are under discussion. This will be denoted by Ω. The primitive notion of set theory is that of belonging. We write $x \in A$ if the object x belongs to the set A.

Suppose that Ω is the set of all character strings. One way to specify a set in Ω is to create a list of elements, such as $Z =$ `{"cat", "dog", "elephant", "ibex"}`. We have `"ibex"` $\in Z$ and `"snark"` $\notin Z$.

Anoter way to specify a set is to use a predicate, as in this example.

$$B = \{s \in \Omega | s[0] = \texttt{b}\}.$$

This specifies all character strings beginning with ‘`b`’.

In this discussion, we will use mathematical notation. Here are its equivalents in various languages.

Symbol	Math	C-family	Python
not	$\sim$	`!`	`not`
and	$\wedge$	`&&`	`and`
or	$\vee$	`||`	`or`
xor	$\oplus$	`-`	`-`

Before we proceed, we must state what it means for one set to equal another. If A and B are sets, we say that A is a *subset* of B if every element of A belongs to B. In this case, we write $A \subseteq B$.

One consequence of this is that duplicate entries in a set are ignored. A set is specified solely by the collection of items in it. We say that

$$A = B$$

if $A \subseteq B$ and $B \subseteq A$. This means that every element of A is an element of B and vice versa.

If A is a set, the we define the *complement* of A to be

$$A^c = \{x \in \Omega | x \notin A\}.$$

To wit, the complement of A is everything in Ω outside of A.

If A and B are sets, we define the *union* of A and B to be

$$A \cup B = \{x \in \Omega | x \in A \vee x \in B\}.$$

This is the set of all elements of Ω belonging to at least one of A or B. We define the `intersection` of A and B to be the set of all elements present in both A and B, i. e.

$$A \cap B = \{x \in \Omega | x \in A \wedge x \in B\}.$$

The *difference* or *relative complemwent* of B in A is defined to be the set of all elements of A not in B. We can write that as

$$A - B = \{x \in \Omega | x \in A \wedge x \notin B\}.$$

The *symmetric difference* of A and B is

$$A \triangle B = \{x \in \Omega | x \in A \oplus x \in B\};$$

this is the set of all things belonging to exactly one of A or B.

Programming Exercises

1. Look in the Python documentation for its `set` type. Figure out how to do unions, intersections, relative complements, and symmetric differences using Python sets.

2. Look in the Java documentation for its `Set<T>` type. Figure out how to do unions, intersections, relative complements, and symmetric differences using Java sets. Note that `Set<T>` is an interface: you must use a `HashSet<T>` or `TreeSet<T>` type for making actual objects.

1 Relations

This concept is important because it is integral to the idea of comparing and ordering objects. At first it seems abstract, but we will see that it has a familiar guise fromm your Miss Wormwood days.

An *ordered pair* from a set S is just a tuple of two elements from S. The set of all ordered pairs in S is also known as the `Cartesian product` of S with itself. We have

$$S \times S = \{(a, b) | a, b \in S\}.$$

A *relation* R on a set S is a subset of $S \times S$. We will use the notation xRy if $(x, y) \in R$ and say that x i is `related to` y. We begin by describing some properties of relations.

- **reflexivity** A relation R on a set S is *reflexive* if every element of S is related to itself. To wit, for all $x \in S$, xRx. is *irreflexive* if every element of S is related to itself. To wit, for all $x \in S$, $\sim (xRx)$.
- **symmetric** A relation R on a set S is *symmetric* if every for all elements $x, y \in S$, xRy guarantees yRx.
- **antsymmetric** A relation R on a set S is *antisymmetric* if every for all elements $x, y \in S$, xRy precludes yRx.
- **transitive** A relation R on a set S is *transitive* if every for all elements $x, y, z \in S$, xRy and yRz guarantees xRz. To wit, for all $x \in S$, xRx.