THE OFFICIAL HANDY–DANDY
ACADEMY MATH CHEAT SHEET

Rules of Exponents
For any nonzero x, $x^0 = 1$.
For any integers p and q,

$$x^{\frac{p}{q}} = \sqrt[q]{x^p} = (\sqrt[q]{x})^p.$$

If p is positive this is defined for all x when q is odd and for nonnegative x when q is even. If p/q is negative, the power $x^{\frac{p}{q}}$ is never defined for $x = 0$.

Other exponent rules include

$$x^{r+s} = x^r x^s \qquad (x^r)^s = x^{rs}$$

$$(xy)^r = x^r y^r \qquad \left(\frac{x}{y}\right)^r = \frac{x^r}{y^r}$$

$$x^{-r} = \frac{1}{x^r}$$

Rules of Logarithms
For any positive numbers, the following hold. Two caveats: $\log_c(x)$ is only defined for $x > 0$ and $c \neq 1$, $c > 0$.

$$\log_a(b) = x \qquad \Longleftrightarrow \qquad a^x = b$$

$$\log_c(xy) = \log_c(x) + \log_c(y)$$

$$\log_c\left(\frac{x}{y}\right) = \log_c(x) - \log_c(y)$$

$$\log_c(x^y) = y \log_c(x)$$

For any bases a and b we have

$$\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$$

The Binomial Theorem
If n is a positive integer and a and b are any numbers,

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}.$$

Recall that

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}.$$

Trig Identities

$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$

The Simplest Identity:

$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$

We hav

$$\theta) = \frac{1}{\sin(\theta)} \qquad \sec(\theta) = \frac{1}{\cos(\theta)} \qquad \cot(\theta) = \frac{1}{\tan(\theta)}$$

The Pythagorean identities are:

$$\begin{aligned} \sin^2(\theta) + \cos^2(\theta) &= 1 \\ \tan^2(\theta) + 1 &= \sec^2(\theta) \\ 1 + \cot^2(\theta) &= \csc^2(\theta) \end{aligned}$$

The addition formulae:

$$\begin{aligned} \sin(\theta + \psi) &= \sin(\theta)\cos(\psi) + \cos(\theta)\sin(\psi) \\ \cos(\theta + \psi) &= \cos(\theta)\cos(\psi) - \sin(\theta)\sin(\psi) \\ \tan(\theta + \psi) &= \frac{\tan(\theta) + \tan(\psi)}{1 - \tan(\theta)\tan(\psi)} \end{aligned}$$

The subtraction formulae:

$$\begin{aligned} \sin(\theta - \psi) &= \sin(\theta)\cos(\psi) - \cos(\theta)\sin(\psi) \\ \cos(\theta - \psi) &= \cos(\theta)\cos(\psi) + \sin(\theta)\sin(\psi) \\ \tan(\theta - \psi) &= \frac{\tan(\theta) - \tan(\psi)}{1 + \tan(\theta)\tan(\psi)} \end{aligned}$$

The double angle formulae:

$$\begin{aligned} \sin(2\theta) & \qquad \theta)\cos(\theta) \\ \cos(2\theta) &= \cos^2(\theta) - \sin^2(\theta) \\ &= 2\cos^2(\theta) - 1 \\ &= 1 - 2\sin^2(\theta) \end{aligned}$$

The half–angle formulas insist *you* keep track of the quadrant:

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 - \cos(\theta)}{2}}$$

$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 + \cos(\theta)}{2}}$$

$$\tan\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 - \cos(\theta)}{1 + \cos(\theta)}}.$$

Domains of Inverse Trig Functions

Function	Domain	Range
arcsin	$[-1, 1]$	$[-\pi/2, \pi/2]$
arccos	$[-1, 1]$	$[0, \pi]$
arctan	all real numbers	$[-\pi/2, \pi/2]$
arctan	all real numbers	$[0, \pi]$
arcsec	$(-\infty, -1] \cup [1, \infty)$	$[0, \pi]$
arccsc	$(-\infty, -1] \cup [1, \infty)$	$[-\pi/2, \pi/2]$

Odd and Even

Odd	Even
sin	cos
tan	sec
cot	
csc	

Formal Rules of Differentiation

Function	Derivative
$f + g$	$f' + g'$
cf	cf'
fg	$f'g + fg'$
$\frac{f}{g}$	$\frac{f'g - fg'}{g^2}$
$f \circ g$	$f' \circ g \cdot g'$
f^n	$nf^{n-1}f'$

Taylor Polynomial If f is a function, its nth degree Taylor polynomial centered at $x = a$ is

$$p_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x - a)^k$$

Newton's Law of Cooling The rate of cooling of a body is proportional to the difference between the ambient temperature and the temperature of the body.

Newton's Method Let f be a differentiable function. Assume you have a guess for the root x_n where $f(x) = 0$. Then you get a new guess x_{n+1} by computing the x-intercept of the tangent line to the graph of $y = f(x)$ at the point $(x_n, f(x_n))$.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

A Brief Derivative Table

f	f'
x^r	rx^{r-1}
a^x	$a^x \ln(a)$
e^x	e^x
$\log_a(x)$	$\frac{1}{x \ln(a)}$
$\ln(x)$	$\frac{1}{x}$
$\sin(x)$	$\cos(x)$
$\cos(x)$	$-\sin(x)$
$\tan(x)$	$\sec^2(x)$
$\cot(x)$	$-\csc^2(x)$
$\sec(x)$	$\sec(x)\tan(x)$
$\csc(x)$	$-\csc(x)\cot(x)$
$\arcsin(x)$	$\frac{1}{\sqrt{1-x^2}}$
$\arccos(x)$	$-\frac{1}{\sqrt{1-x^2}}$
$\arctan(x)$	$\frac{1}{1+x^2}$
$\operatorname{arccot}(x)$	$-\frac{1}{1+x^2}$
$\operatorname{arcsec}(x)$	$\frac{1}{\lvert x\rvert\sqrt{x^2-1}}, \qquad \lvert x\rvert > 1$
$\operatorname{arccsc}(x)$	$-\frac{1}{\lvert x\rvert\sqrt{x^2-1}}, \qquad \lvert x\rvert > 1$