Solutions to Problem Set 2, Discrete G Tri 1, 0405

1. Show that $(A \cap B)^c = A^c \cup B^c$ implies $(A \cup B)^c = A^c \cap B^c$.

Solution. In the identity $(A \cap B)^c = A^c \cup B^c$, replace A with A^c and B with B^c to get

$$(A^c \cap B^c)^c = A^{cc} \cup B^{cc} = A \cup B.$$

To finish, take compliments on both sides.

2. Prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

Solution. To do this we will establish the tautology

$$P \vee (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \vee R).$$

This can be done with the following truth table

P	Q	R	$P \wedge Q$	$P \wedge R$	$(Q \vee R)$	$P \wedge (Q \wedge R)$	$(P \wedge Q) \vee (P \wedge R)$
T	T	T	T	T	T	T	T
T	T	F	T	F	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	F	F	F	F
F	T	T	F	F	T	F	F
F	T	F	F	F	T	F	F
F	F	T	F	F	T	F	F
F	F	F	F	F	F	F	F

3. Prove that $A \Delta B = (A \cup B) - (A \cap B)$.

Solution.

We shall invoke the DeMorgan and distributive laws in the following chain of equalities

$$\begin{aligned}
(A \cup B) - (A \cap B) &= (A \cup B) \cap (A \cap B)^c \\
&= (A \cup B) \cap (A^c \cup B^c) \\
&= (A \cap A^c) \cup (A \cap B^c) \cup (B \cap B^c) \cup (A^c \cap B) \\
&= \emptyset \cup (A - B) \cup \emptyset \cup (B - A) \\
&= (A - B) \cup (B - A) = A \Delta B.
\end{aligned}$$