Homework 3
An Assorted Box of Combinatorial Bonbons
discreteG, 0405, T1

Make sure you write a *careful* explanation of your solution in complete sentences!

1. How many strings of three or fewer characters from the alphabet $\Sigma = \{a, e, i, o, u\}$ can be created? How many of these have all different letters in them?

2. A faculty has 70 members. Four committees are to be created: the academic technology committee, the graduation requirements committee, the academic affairs committee and the doughnut tasting committee. Each committee is to have six members. How many possible assignments exist? How many ways are there to create four teams of six to distribute to the committees?

3. A lazy Susan is to hold 12 different bottles of spices. How many distinguishable ways are there to order the spices on the lazy Susan.? Remember, if one arrangement can be rotated to another, it is considered to be the same.

4. Develop a general principle from the last problem. What do you see?

5. Use induction to prove that $\binom{n}{2}$ is even if n is even. What about the converse? What can you say about the evenness or oddness of $\binom{n}{k}$ in terms of that of n or k? If that fails, can you say anything else??

6. North Carolina license plates consist of three letters followed by 2, 3 or 4 numbers. The letters O, Q are omitted. Tell how many are possible altogether. Research this question and argue one way or the other: Does it seem like a sufficient supply for the foreseeable future? Substantiate your argument with facts!

7. Suppose that f is a function and that D is an operator in the sense that if f is a function, $D(f)$ is another function. Suppose that D is a *derivation*, i.e. that

$$D(f \cdot g) = f \cdot D(g) + g \cdot D(f),$$

for any functions f and g. Now if n is a positive integer let $D^{(n)}(f)$ denote the application of D to f n times. F'rinstance, $D^{(2)}(f) = D(D(f))$. Find a rule for $D^{(n)}(f \cdot g)$, where f and g are functions.

8. Find a formula for

$$\sum_{k=0}^{n} \binom{n}{k}^2 .$$

Prove this with induction. To help you see a pattern write out several (I might suggest 10) rows of Pascal's triangle and look around for a pattern.

9. How many different words of length six can be constructed from the word BAFFLEGAB?

10. We all know that there are $\binom{52}{5}$ poker hands. Tell how many of these hands have a *full house*, i.e. a hand with three cards of one rank and two of another? How likely would you say a full house on the deal is?